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GRADIENT METHOD OF OPTIMAL CONTROL APPLIED TO THE OPERATION OF A DAM WATER GATE

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SUMMARY

An extension of the authors' previous methods is presented for the optimal control of flood propagation via a dam gate, based on a combination of the finite element and gradient methods. It is assumed in previous papers that the control duration is the same as the duration of the flood. However, the duration of the control does not necessarily coincide with that of the flood flow. To overcome this difficulty, the gradient method is applied to solve the free terminal time–fixed terminal condition problem. It is shown that the water elevation can be controlled exactly the same as with the previously presented method. It is also shown that the computation can be terminated at a far shorter time than the terminal time of the flood.

KEY WORDS Water gate of dam Gradient method Optimal control Finite element method Hydraulic model

1. INTRODUCTION

Previous studies on dam gate control have all been conducted without considering a hydraulic model.[1–10] Recently Kawahara and his group presented optimal control methods to obtain strategies to control dam water gates for the prevention of flood flow by considering a hydraulic model based on the shallow water equation.[11–16] The model is introduced to express the propagation of a wave generated by the sudden operation of a dam gate. For instance, the generation of a reflective wave towards the upstream area through a reservoir causes tremendous damage along the coast of the upstream area. Such phenomena must be described by a hydraulic model. To obtain useful information on dam gate control to solve such problems, Kawahara and his group presented optimal control methods based on tracking control techniques and the finite element method.[11–14] The numerical examples given in these papers show that the water elevation of the reservoir was well controlled by the tracking control method. However, this method requires the whole pattern of the flood to be described as a time function whose magnitude and duration are known. This means that the control of the dam gate can only start after the flood has passed through the reservoir. To overcome this impractical defect, the authors presented a predictive control method which was shown to be useful in practice.[15,16]

The previous control methods assume that the starting and final times of the control correspond to those of the flood flow. However, it is not necessarily the case that the final time of the control is the same as the final time of the flood. The final time of the control may be longer than that of the flood in which case the control is not satisfactory. If the final time of the control can be shortened, this will save computational effort. In this case the final time of the control is unknown and must be determined by the condition that the water elevation

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reached at the final time is the still water level. This type of control is referred to as the free terminal time–fixed terminal condition problem.

This paper presents an optimal control method using a combination of the finite element and gradient methods, which as utilized in this paper is described as follows. The optimal control problem can be defined as one of finding an optimal control vector so as to minimize the performance function within the constraint of the state equation. The terminal time of the control in this case is unknown and must be determined by the terminal condition. Thus an iteration procedure is required to solve the problem. At this state the variations in the control vector and terminal time must be defined and determined. The variation in the terminal condition can be formulated in a useful form by introducing an adjoint function which is a function of the control vector, terminal time and terminal condition. Assuming a small value of the variation in the terminal condition, the variations in the control vector and terminal time are computed. Then, by comparing the newly obtained and previously assumed variations in the terminal condition, the iteration procedure may be continued. The control vector and terminal time can be obtained if their variations are computed to be small enough to satisfy the terminal condition.

In the present method only the linear shallow water equation is used as the governing equation. The extension to the non-linear equation is straightforward. The two-step explicit finite element method is used to solve the governing equation, Euler–Lagrange equation and adjoint equation. To show the adaptability of the present method, several numerical examples are computed. The method presented in this paper is shown to be useful to solve the free terminal time–fixed terminal condition problem.

2. BASIC EQUATIONS

Wave propagation through a reservoir made by a dam can be expressed by the two-dimensional shallow water equation. The linearized shallow water equation without viscosity and friction is used because of its simplicity. The purpose of this paper is to show the strategy of operation of the water gate of the dam. Therefore it is not necessary to use a complicated equation. The effect of non-linearity has already been discussed in Reference 14. Let t be time and x_i $(i = 1,2)$ be a Cartesian co-ordinate placed at the still water level as shown in Figure 1. The origin of the co-ordinate is placed at the top of the water gate.

Denoting the mean discharge and water elevation as q_i and ζ respectively, the equations of motion and continuity can be written as

$$\dot{q}_i + gh\zeta_{,i} = 0 \quad \text{in } \Omega, \tag{1}$$

$$\dot{\zeta} + q_{i,i} = 0 \quad \text{in } \Omega, \tag{2}$$

where Ω represents the whole domain and g and h are the gravitational acceleration and water depth respectively. The flood wave is characterized by the water discharge at the inlet S_f as

$$q_i = \hat{q}_i \quad \text{on } S_f \quad [t_0, t_f], \tag{3}$$

where $\hat{q}_i$ is a hydrograph of the flood and is a prescribed function of time t. This is given by the observation of the flood at the upstream site.

The water elevation is controlled by the outflow discharge $\bar{q}_i$ which is calculated by the control method. The mean outflow discharge $\bar{q}_i$ is specified on the outlet boundary S_u as

$$q_i = \bar{q}_i \quad \text{on } S_u \quad [t_0, t_f], \tag{4}$$

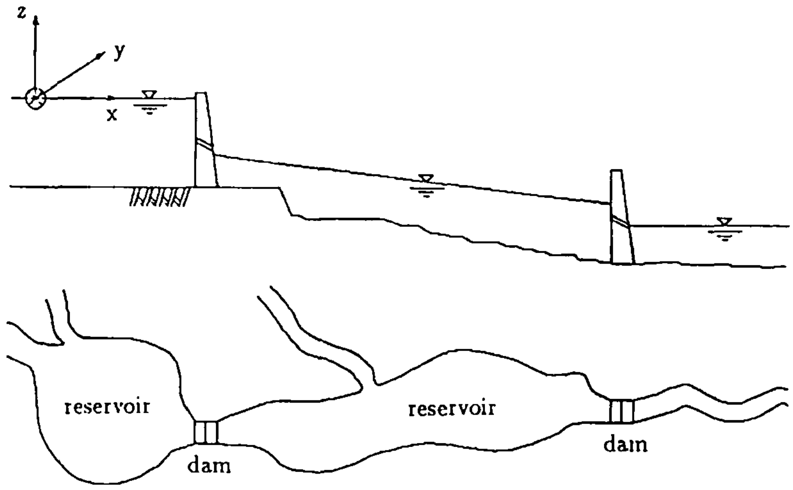


Figure 1. Dam model

where $\bar{q}_i$ is a function of time to be determined by the optimal theory. The restriction condition of the water gate must be imposed on the outflow discharge. The precise formulation is discussed in Reference 17.

The initial conditions are given as

$$\zeta = \hat{\zeta}^0 \quad \text{at } t = t_0, \tag{5}$$

$$q_i = \hat{q}_i^0 \quad \text{at } t = t_0. \tag{6}$$

The optimal control is formulated to find the function $\bar{q}_i$ such that the performance function

$$J = \frac{1}{2}\int_{t_0}^{T_f}\int_{\Omega} (Q\zeta^2 + R_{ij}\bar{q}_i\bar{q}_j)\, \mathrm{d}\Omega\, \mathrm{d}t \tag{7}$$

is minimized through the time duration $[t_0, T_f]$. In equation (7) Q and R_{ij} are weighting functions which determine the adjustment of the difference in dimension between ζ and $\bar{q}_i$. The final time T_f of the control is not necessarily coincident with the final time t_f of the flood. Thus the final time T_f is unknown but can be determined by the condition that the final water elevation and water discharge should be zero as soon as possible, i.e. the equation

$$N(\zeta(T_f), q_i(T_f), T_f) = \frac{s}{2}\zeta^{\mathrm{T}}\zeta + \frac{w}{2}\bar{q}_i^{\mathrm{T}}\bar{q}_i = 0 \tag{8}$$

must be satisfied, where s and w are constants. In this paper the optimal control expressed by the function J in equation (7) is discussed. Thus the strategy of operation of the water gate of the dam can be obtained based on the minimization of the function J.

The optimal control strategy presented in this paper is to find $\bar{q}_i$ on S_u that minimizes the performance function J under the constraints of (1)–(4), initial conditions (5) and (6) and final condition (8). The duration of the control, T_f can be shorter than the duration of the flood, t_f. If the control computation is limited by T_f, the running time of computation can be intensively saved.

3. FINITE ELEMENT EQUATION

The weighted residual equations are obtained by employing the conventional Galerkin method applied to the governing equations. The standard finite element procedure based on the linear triangular element leads to the equation

$$[M]\{\dot{x}\} + [H]\{x\} = 0, \tag{9}$$

where $\{x\}$ represents the water elevation and velocity at nodal points of the flow field. This is referred to as the state vector and

$$[M] = \begin{bmatrix} [M_{\alpha\beta}] & & \\ & [M_{\alpha\beta}] & \\ & & [M_{\alpha\beta}] \end{bmatrix}, \qquad H = \begin{bmatrix} & & gh[S_x] \\ & & gh[S_y] \\ [S_x] & [S_y] & \end{bmatrix},$$

$$M_{\alpha\beta} = \int_\Omega (\Phi_\alpha \Phi_\beta)\, d\Omega, \qquad S_{\alpha\beta i} = \int_\Omega (\Phi_\alpha \Phi_{\beta,i})\, d\Omega,$$

in which Φ_α is the interpolation function for both discharge and water elevation.

The state equation can be obtained in the following form by separating the terms of control discharge and flood discharge:

$$\begin{aligned}\{\dot{x}\} &= -[M]^{-1}[H]\{x\} \\ &= [A]\{x\} + [B]\{u\} + [C]\{f\},\end{aligned} \tag{10}$$

where $\{x\}$ in the second line designates the discharge and water elevation at nodal points except those located on S_u and S_f, $\{u\}$ and $\{f\}$ are nodal values of control discharge $\bar{q}_i$ and flood discharge $\hat{q}_i$ respectively and $[A]$, $[B]$ and $[C]$ are the corresponding coefficient matrices.

For the numerical integration in time of the state equation (10), a time-marching numerical integration scheme is used. The total time interval to be analysed is divided into a large number of short time intervals, one of which is denoted by Δt. Representing the time point by n, the two-step explicit method can be applied to equation (10): for the first step

$$\{x_{n+1/2}\} = [\bar{M}]^{-1}[\tilde{M}]\{x_n\} - \frac{\Delta t}{2}[\bar{M}]^{-1}[H]\{x_n\} \tag{11}$$

and for the second step

$$\{x_{n+1}\} = [\bar{M}]^{-1}[\tilde{M}]\{x_n\} - \Delta t[\bar{M}]^{-1}[H]\{x_{n+1/2}\}, \tag{12}$$

starting from the initial condition equations (5) and (6). In equations (11) and (12) the lumped coefficient matrix $[\bar{M}]$ is introduced to obtain the full explicit scheme. To secure stability, the mixed coefficient matrix $[\tilde{M}]$ is used as

$$[\tilde{M}] = e[\bar{M}] + (1 - e)[M], \tag{13}$$

where e is referred to as the lumping parameter.

The optimal control problem is formulated so as to determine $\{u\}$ for the control function to minimize the performance function

$$
\begin{aligned}
J &= \int_{t_0}^{T_f} f_0(x, u, t)\, dt \\
&= \tfrac{1}{2}\int_{t_0}^{T_f} (\{\zeta\}^T[S']\{\zeta\} + \{u\}^T[R]\{u\})\, dt \\
&= \tfrac{1}{2}\int_{t_0}^{T_f} (\{x\}^T[S]\{x\} + \{u\}^T[R]\{u\})\, dt
\end{aligned} \tag{14}
$$

under the state equation with the initial condition $\{x_0\}$, where $[S]$ and $[R]$ are weighting matrices and t_0 and T_f are the initial and final times respectively of the time domain. The final time T_f is unknown and is decided on the condition that

$$N(x(T_f), T_f) = \tfrac{1}{2}(\{x(T_f)\}^T[S]\{x(T_f)\} + \{u(T_f)\}^T[R]\{u(T_f)\}) = 0. \tag{15}$$

For the optimization technique to seek the minimum value of J in equation (14) with the state equation as constraints, the gradient method is used to search for the minimum value of J along the direction of the gradient of J.

4. GRADIENT METHOD

To explain the optimal control method employed in this paper, it is worthwhile to summarize the basic formulation of the problem again. The problem treated in this paper can be classified as the so-called free terminal time–fixed end condition problem. The state vector $\{x\}$ is defined as

$$
\begin{aligned}
\{\dot{x}\} &= F(x, u, t) \\
&= [A]\{x\} + [B]\{u\} + [C]\{f\},
\end{aligned} \tag{16}
$$

with the initial condition

$$\{x(t_0)\} = \{x^0\}.$$

Find the control vector $\{u\}$ so as to minimize the performance function

$$J = \tfrac{1}{2}\int_{t_0}^{T_f} (\{x\}^T[S]\{x\} + \{u\}^T[R]\{u\})\, dt, \tag{17}$$

where T_f is the terminal time, which is unknown and is determined by the terminal condition

$$N(x(T_f), T_f) = 0. \tag{18}$$

It is well known that the minimizing function $\{u\}$ for J can be obtained by introducing the Hamiltonian function

$$H = \tfrac{1}{2}\{x\}^T[S]\{x\} + \tfrac{1}{2}\{u\}^T[R]\{u\} + \{p\}^T([A]\{x\} + [B]\{u\} + [C]\{f\}), \tag{19}$$

where $\{p\}$ denotes a Lagrange function. The Euler–Lagrange equation and transversality condition can be described as

$$\{\dot{p}\} = -\frac{\partial H}{\partial\{x\}} = -[S]\{x\} - [A]^{\mathrm{T}}\{p\}, \tag{20}$$

$$\{p(T_f)\} = \{0\}. \tag{21}$$

The control vector $\{u\}$ can be obtained based on $\{p\}$, but this is impossible at this stage because the terminal time T_f is not yet determined.

To solve the problem, consider small variations in the control vector, $\{\delta u\}$, and terminal time, δT_f, assuming the existence of T_f. Introduce the adjoint vector $\{z\}$ which is defined by

$$\{\dot{z}\} = -\left(\frac{\partial F}{\partial x}\right)^{\mathrm{T}}\{z\} = -[A]^{\mathrm{T}}\{z\}, \tag{22}$$

$$\{z(T_f)\} = \frac{\partial N(x(T_f), T_f)}{\partial x} = [S]\{x(T_f)\}. \tag{23}$$

The variation in $N(x(T_f),T_f)$ for the variations in the state vector, $\{\delta x\}$, and terminal time, δT_f, can be written in the form

$$\delta N = \left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_f}\delta T_f + \int_{t_0}^{T_f}\{z\}^{\mathrm{T}}\frac{\partial N}{\partial x}\,\delta x(T_f)\,\mathrm{d}t, \tag{24}$$

where

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\partial N}{\partial t} + \left(\frac{\partial N}{\partial x}\right)^{\mathrm{T}}\{\dot{x}\}. \tag{25}$$

After some calculation the second term of equation (24) can be described as

$$\delta N = \left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_f}\delta T_f + \int_{t_0}^{T_f}\{z\}^{\mathrm{T}}\frac{\partial F}{\partial u}\,\delta u\,\mathrm{d}t, \tag{26}$$

where

$$\frac{\partial F}{\partial u} = [B]. \tag{27}$$

The variation in the performance function J can be obtained in the form

$$\delta J = f_0(x(T_f), u(T_f), T_f)\delta T_f + \int_{t_0}^{T_f}\left(\frac{\partial H}{\partial u}\right)^{\mathrm{T}}\delta u\,\mathrm{d}t. \tag{28}$$

5. SOLUTION PROCEDURE OF GRADIENT METHOD

To solve the optimal control problem, the iteration method can be applied based on $\{\delta u\}$ and δT_f. For this purpose a new performance function is introduced for $\{\delta u\}$ and δT_f considering the penalty of the restrictive condition equation (26):

$$\delta J = \delta J + \tfrac{1}{2}\alpha(\delta T_f)^2 + \int_{t_0}^{T_f}\tfrac{1}{2}(\delta u)^{\mathrm{T}}[W]\delta u\,\mathrm{d}t + \{v\}^{\mathrm{T}}\left[\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_f}\delta T_f + \int_{t_0}^{T_f}\{z\}^{\mathrm{T}}\frac{\partial F}{\partial u}\,\delta u\,\mathrm{d}t - \delta N\right], \tag{29}$$

where $\{v\}$ is the Lagrange multiplier, α is a weighting value and $[W]$ is a weighting matrix.

The first variation in the new performance function can be obtained in the form

$$\delta(\delta\bar{J}) = \left(f_0 + \{v\}^{\mathrm{T}}\frac{\mathrm{d}N}{\mathrm{d}t} + \alpha\delta T_{\mathrm{f}}\right)_{T_{\mathrm{f}}}\delta(\delta T_{\mathrm{f}}) + \int_{t_0}^{T_{\mathrm{f}}}\left[\left(\frac{\partial H}{\partial u}\right)^{\mathrm{T}} + \{v\}^{\mathrm{T}}\{z\}^{\mathrm{T}}\frac{\partial F}{\partial u} + (\delta u)^{\mathrm{T}}[W]\right]\delta(\delta u)\,\mathrm{d}t, \tag{30}$$

Seeking the minimum value of the new performance function, $\{\delta u\}$ and δT_{f} can be derived as

$$\begin{aligned}\delta u(t) &= -[W]^{-1}\left[\frac{f_0(x(t), u(t), t)}{\partial u} + \left(\frac{F(x(t), u(t), t)}{\partial u}\right)^{\mathrm{T}}(\{p\} + \{v\}^{\mathrm{T}}\{z\})\right]\\ &= -[W]^{-1}[[R]\{u\} + [B]^{\mathrm{T}}(\{p\} + \{v\}\{z\})],\end{aligned} \tag{31}$$

$$\begin{aligned}\delta T_{\mathrm{f}} &= -\frac{1}{\alpha}\left(f_0 + \{v\}^{\mathrm{T}}\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}}\\ &= -\frac{1}{\alpha}\left(\tfrac{1}{2}(\{x\}^{\mathrm{T}}[S]\{x\} + \{u\}^{\mathrm{T}}[R]\{u\}) + \{v\}\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}},\end{aligned} \tag{32}$$

where $\{\delta u\}$ is the difference between the previous control function and the new control function, which is expected to be an improved function, and δT_{f} is the difference between the previously assumed terminal time of control duration and the newly obtained terminal time.

Substituting $\{\delta u\}$ and δT_{f} into equation (26), the variation in $N(x(T_{\mathrm{f}}), T_{\mathrm{f}})$ can be described as

$$\delta N = -\frac{1}{\alpha}\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}}\left(f_0 + \{v\}^{\mathrm{T}}\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}} - (I_{NN}\{v\} + I_{NJ}), \tag{33}$$

where

$$\begin{aligned}I_{NN} &= \int_{t_0}^{T_{\mathrm{f}}}\{z\}^{\mathrm{T}}\frac{\partial F}{\partial u}[W]^{-1}\left(\frac{\partial F}{\partial u}\right)^{\mathrm{T}}\{z\}\,\mathrm{d}t\\ &= \int_{t_0}^{T_{\mathrm{f}}}\{z\}^{\mathrm{T}}[B][W]^{-1}[B]^{\mathrm{T}}\{z\}\,\mathrm{d}t,\end{aligned} \tag{34}$$

$$\begin{aligned}I_{NJ} &= \int_{t_0}^{T_{\mathrm{f}}}\{z\}^{\mathrm{T}}\frac{\partial F}{\partial u}[W]^{-1}\frac{\partial H}{\partial u}\,\mathrm{d}t\\ &= \int_{t_0}^{T_{\mathrm{f}}}\{z\}^{\mathrm{T}}[B][W]^{-1}([R]\{u\} + [B]^{\mathrm{T}}\{p\})\,\mathrm{d}t.\end{aligned} \tag{35}$$

The Lagrange multiplier $\{v\}$ can be obtained in the form

$$\{v\} = -\left[I_{NN} + \frac{1}{\alpha}\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}}\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}}^{\mathrm{T}}\right]^{-1}\left[\delta N + I_{NJ} + \frac{1}{\alpha}f_0(x(T_{\mathrm{f}}), u(T_{\mathrm{f}}), T_{\mathrm{f}})\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_{\mathrm{f}}}\right]. \tag{36}$$

Substituting equations (31) and (32) into equation (28), the variation in the performance function δJ can be derived as

$$\delta J = -\frac{1}{\alpha} f_0(x(T_f), u(T_f), T_f)\left(f_0 + \{v\}^{\mathrm{T}} \frac{\mathrm{d}N}{\mathrm{d}t}\right)_{T_f} - (I_{JJ} + I_{NJ}^{\mathrm{T}}\{v\}), \tag{37}$$

where

$$\begin{aligned} I_{JJ} &= \int_{t_0}^{T_f} \{z\}^{\mathrm{T}}\left(\frac{\partial H}{\partial u}\right)^{\mathrm{T}} [W]^{-1} \frac{\partial H}{\partial u}\,\mathrm{d}t \\ &= \int_{t_0}^{T_f} ([R]\{u\} + [B]^{\mathrm{T}}\{p\})^{\mathrm{T}}[W]^{-1}([R]\{u\} + [B]^{\mathrm{T}}\{p\})\,\mathrm{d}t. \end{aligned} \tag{38}$$

The purpose is to find the control function $\{u(t)\}$ and control terminal time T_f which minimize the performance function J and satisfy $N(x(T_f), T_f) = 0$. To do this, the following method is employed in this study. The terminal time T_f is assumed to be obtained from the relation

$$\delta N(x(T_f), T_f) = 0, \tag{39}$$

holding the control function $\{u(t)\}$ constant. Then the control function may be determined to minimize the performance function $\delta \bar{J}$, holding the terminal time T_f constant.

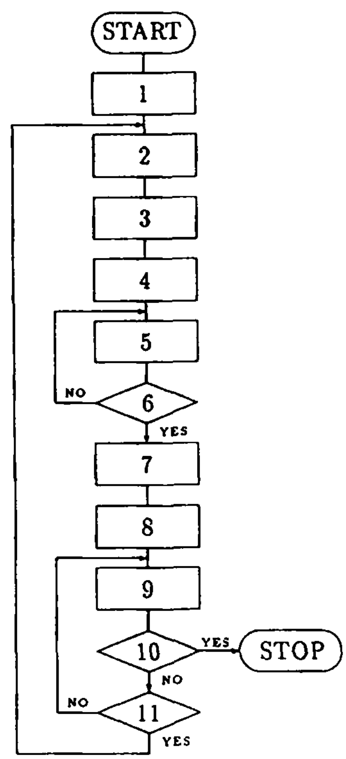
6. COMPUTATIONAL ALGORITHM

The algorithm of the gradient method employed in this paper is described as follows. First, δN is assumed as a small value, e.g.

$$\delta N = \varepsilon_s \cdot N(x(T_f), u(T_f), T_f), \qquad \varepsilon_s = 0{\cdot}1.$$

Secondly, the adjoint function $\{z\}$ can be obtained by solving equation (22) backwards from the final condition equation (23). Thirdly, the functions I_{NN}, I_{NJ}, I_{JJ} and $\{v\}$ can be computed. Fourthly, δN can be computed based on the renewed values using equation (33). Fifthly, by comparing the computed δN with the assumed δN, δT_f can be verified. If δN computed is not close enough to δN assumed, assign the constant α a larger value and recompute. If δN computed is satisfactory, δT_f can be computed using equation (32). Sixthly, using the computed δT_f, the total control time T_f can be determined. Finally, the control function $\{u(t)\}$ can be derived to minimize the performance function J based on the terminal time T_f using equation (31). The algorithm can be described as follows, in which the tolerance ε is a small number.

1. Assume $\{u_0\}$, T_{f0} and set $i = 0$.
2. Obtain $\{x_i\}$ using $\{u_i\}$, T_{fi} by equations (11) and (12) and obtain J_i.
3. Solve Lagrange function $\{p\}$ and adjoint function $\{z\}$: $\{\dot{p}\} = -[S]\{x\} - [A]^T\{p\}$, $\{p(T_{fi})\} = 0$, $\{\dot{z}\} = -[A]^T\{z\}$, $\{z(T_{fi})\} = [S]\{x(T_{fi})\}$.
4. Assume δN^* using $\delta N^* = -\varepsilon_s * N(x(T_{fi}), T_{fi})$ and solve $\{v\}$ by equation (36).
5. Solve $\{\delta u_i\}$ and δT_{fi} using equations (31) and (32).
6. Obtain $\{x_{i+1}\}$, using $\{u_{i+1}\} \simeq T_{fi+1}$.
   IF $\delta N - \delta N^* \geqslant 0$ THEN
   choose larger α ELSE chose smaller α.
   IF $N_i - N_{i+1} \geqslant 0$ THEN GO TO 7 ELSE GO TO 5.
7. Obtain $\{x_{i+1}\}$ using $\{u_{i+1}\}$, T_{fi+1} by equations (11) and (12) and solve $\{p\}$, $\{z\}$.
8. Assume δN^* using $\delta N^* = -\varepsilon_s * N(x(T_{fi+1}), T_{fi+1})$ and solve $\{v\}$ by equation (36).
9. Solve $\{\delta u_i\}$ using equation (31).
10. IF $N(x(T_{fi+1}), T_{fi+1}) < \varepsilon$, $\int_{t_0}^{T_f} (u_{i+1} - u_i)^2 dt < \varepsilon$ THEN STOP ELSE GO TO 11.
11. Obtain $\{x_{i+1}\}$ using $\{u_{i+1}\}$, T_{fi+1} and obtain J_{i+1}.
   IF $J_i - J_{i+1} > 0$ THEN $i = i + 1$ and GO TO 2 ELSE choose larger $[W]$ and GO TO 9.



7. NUMERICAL EXAMPLES

7.1. One-dimensional channel

As a numerical example a simple one-dimensional channel has been computed to test the adaptability of the present method. On the inlet boundary S_f the inflow discharge of the flood assumed is imposed and the normal velocity component is given as zero at the horizontal walls S_c in Figure 2. The inflow discharge is a function of time and is shown in Figure 3(a). The weighting matrices were selected as $R = 0{\cdot}0001$ and $Q =$ unit matrix and the time increment Δt was 0·6 s. The results computed without the control and by the gradient method are compared in Figure 3. The results shown by solid lines are those by the gradient method and the results marked as circles are those without the control. By controlling the discharge at the outlet as shown in Figure 3(c), the water elevation upstream and downstream can be controlled as shown in Figures 3(d)–3(f) at sites A, B and C respectively. It is seen in Figure 3 that an almost flat water elevation has been obtained. The results computed by the tracking control of the Sakawa–Shindo method[14] and by the gradient method are compared in Figure 4. The results

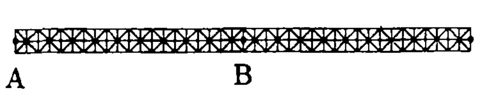
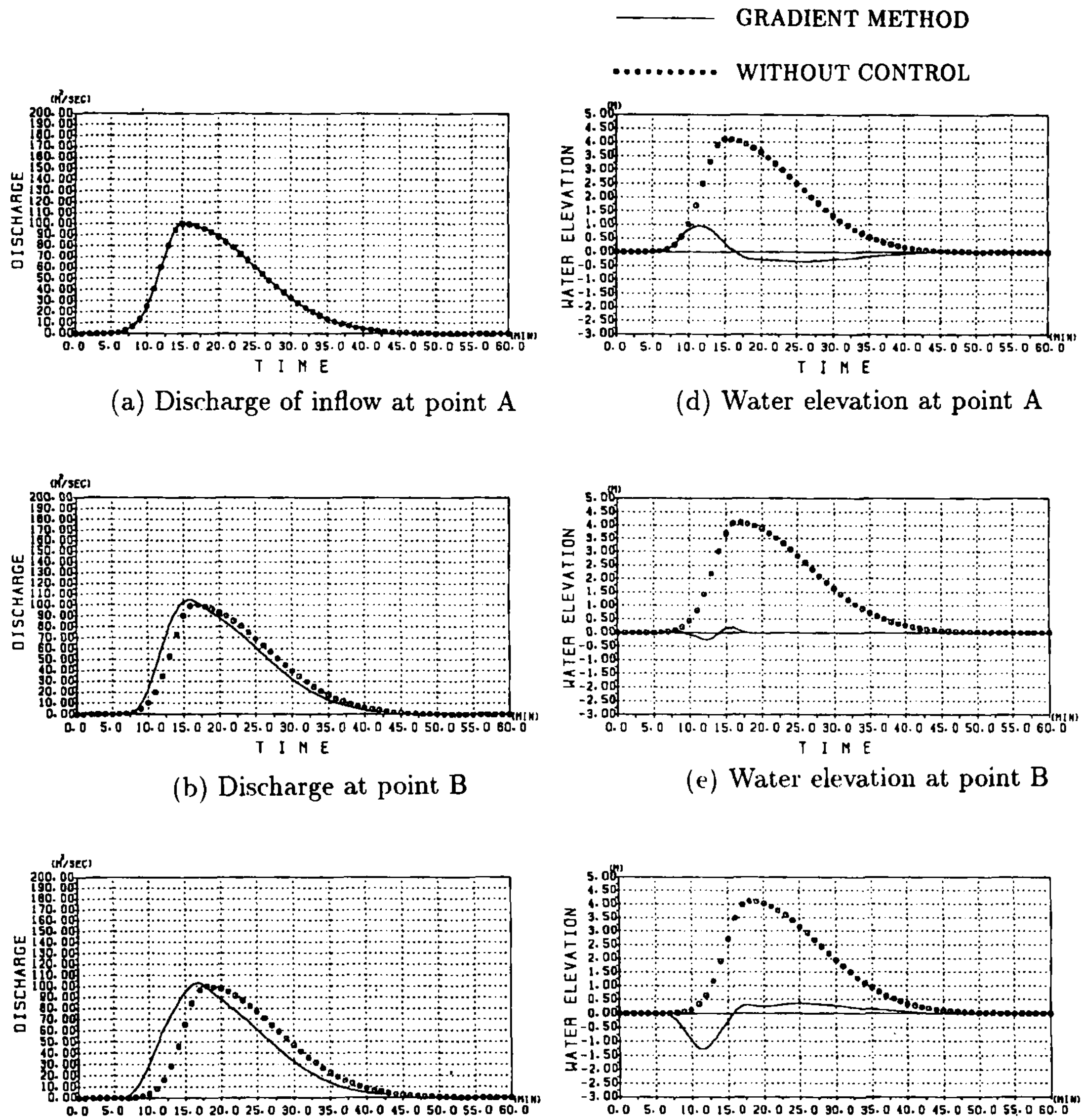


Figure 2. Finite element mesh for one-dimensional channel problem. Length: 4000 m, Width: 200 m, Depth: 60 m, S_f: inlet boundary. S_u: outlet boundary, S_c: boundary condition, NX: 123, MX: 160



(a) Discharge of inflow at point A

(d) Water elevation at point A

(b) Discharge at point B

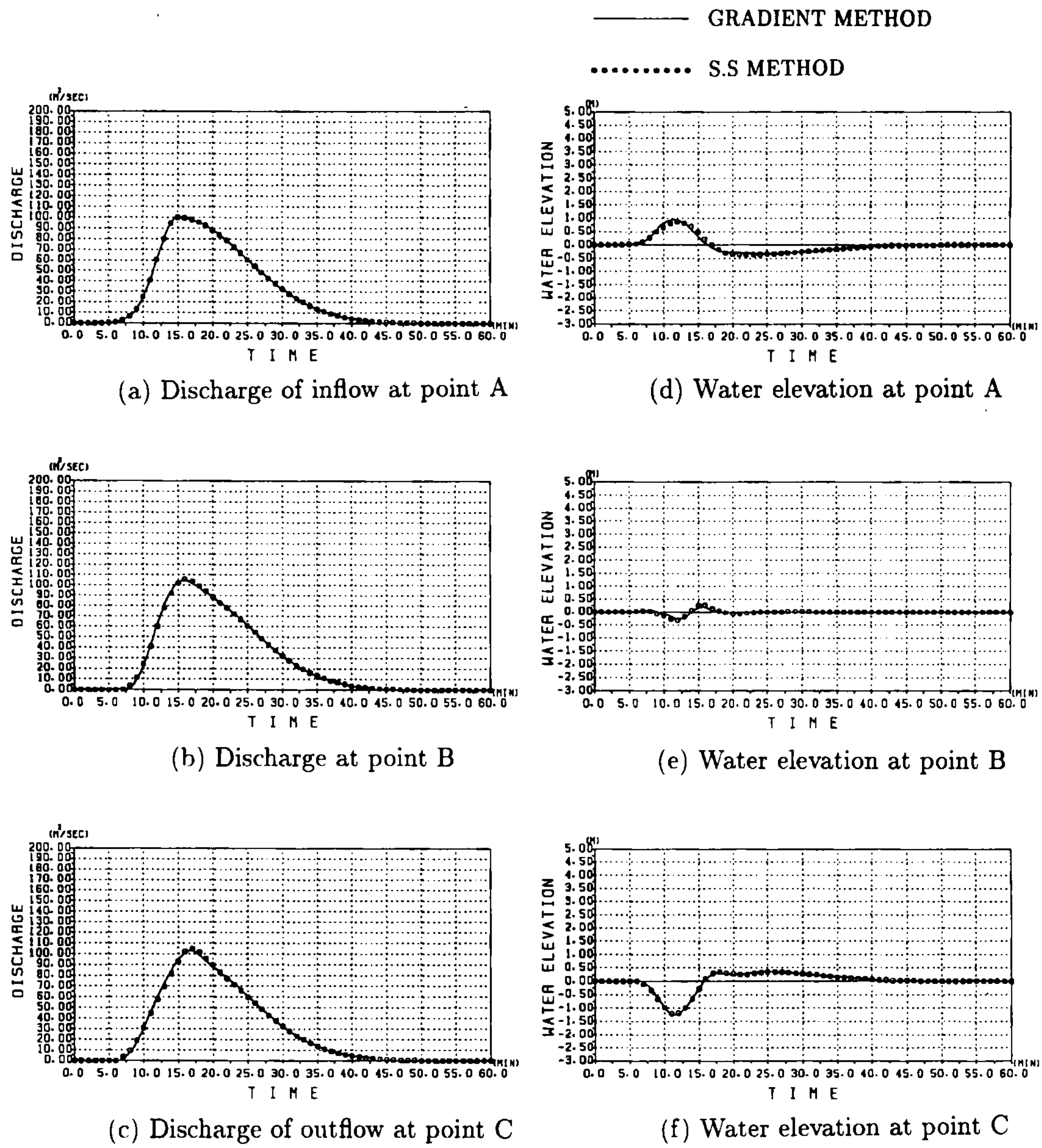
(e) Water elevation at point B

(c) Discharge of outflow at point C

(f) Water elevation at point C

Figure 3

shown by solid lines are those by the gradient method and the results marked as circles are those by the Sakawa–Shindo method. In Figures 4(c) and 4(f) the computed results of the discharge and water elevation respectively at point C obtained by the present method are shown in comparison with the results obtained by the Sakawa–Shindo method in which the duration time of the control (T_f) is assumed to be coincident with the terminal time of the flood (t_f). Both sets of results are completely coincident for both discharge and water elevation.



(a) Discharge of inflow at point A

(d) Water elevation at point A

(b) Discharge at point B

(e) Water elevation at point B

(c) Discharge of outflow at point C

(f) Water elevation at point C

Figure 4

7.2. Control of Moriyoshizan dam gate

The Moriyoshizan dam is constructed to reserve water for irrigation purposes. The dam is located at site C in Figure 5. The dam is 80 m high and 200 m long. A water gate is provided at the crest of the dam. The reservoir is about 5 km long and 700 m wide. The average water depth is 60 m. The main flood flows from the Moriyoshi river to the reservoir; the flow point is indicated as site A in Figure 5. A typical hydrograph is shown in Figure 6(a).

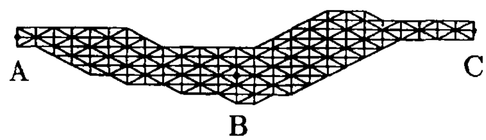


Figure 5. Finite element mesh for Moriyoshizan dam problem

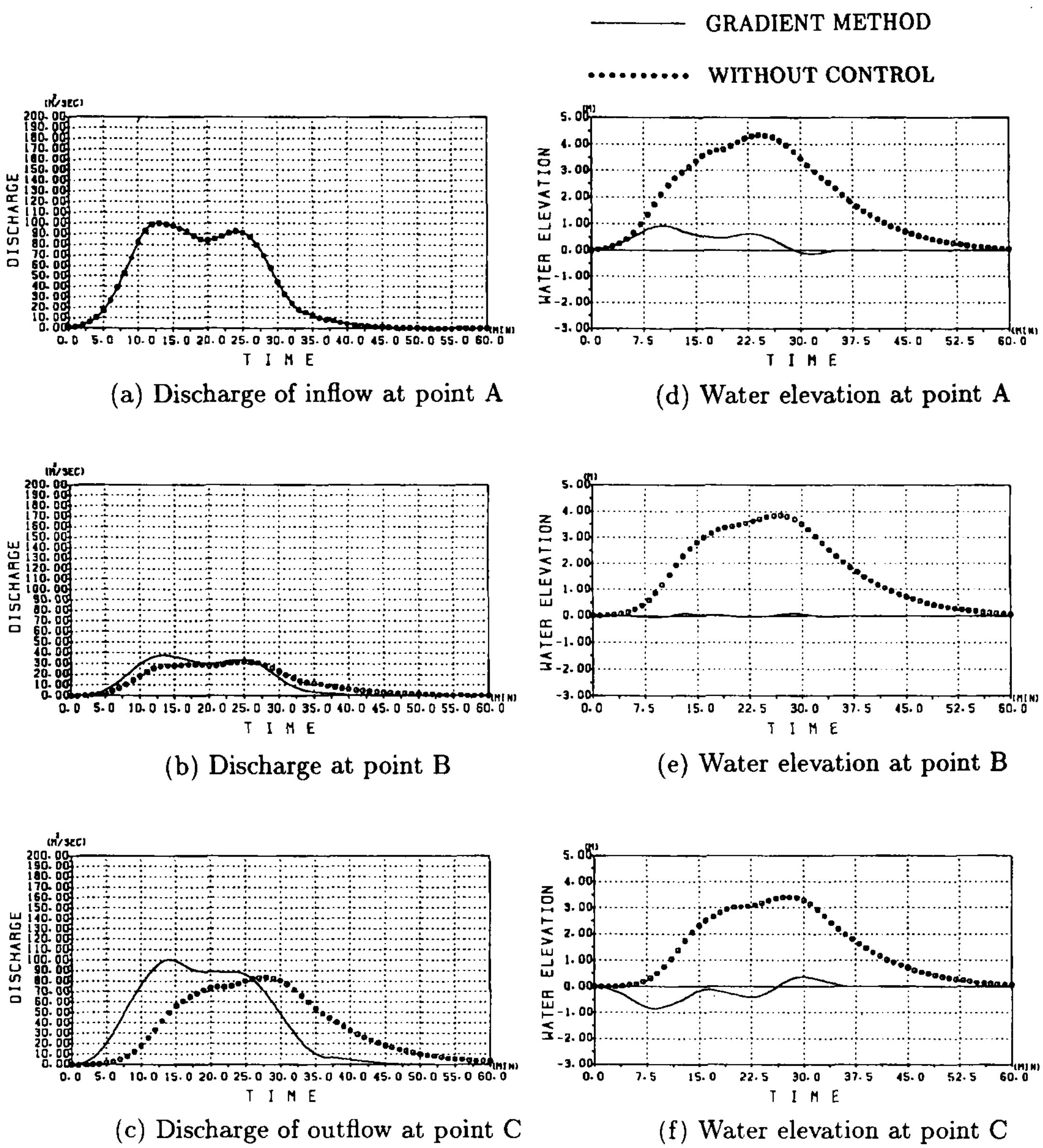
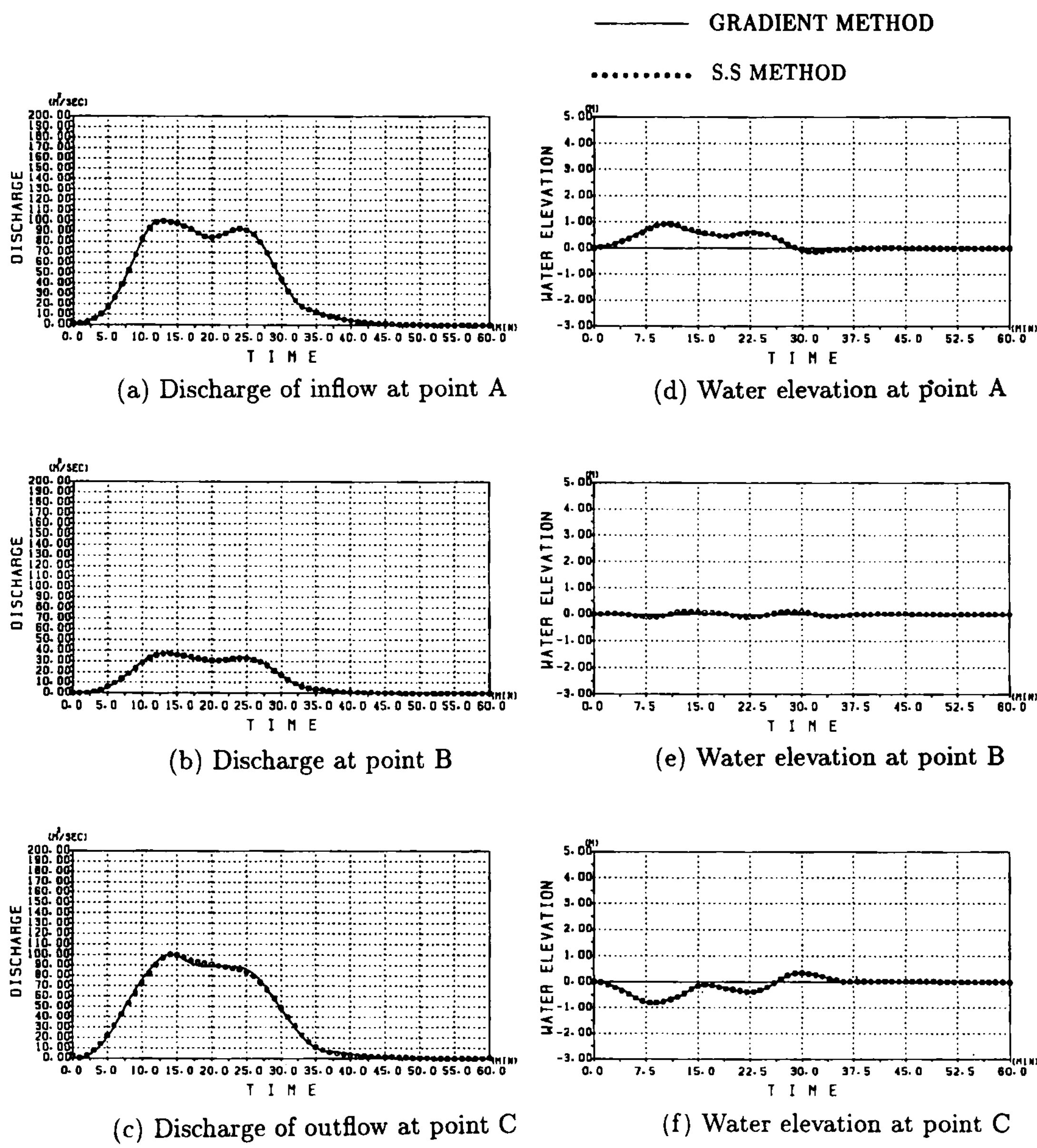


Figure 6.

The computation is carried out via the gradient control method based on the finite element idealization represented in Figure 6. The total numbers of finite elements and nodal points are 140 and 224 respectively. The weighting matrices selected were $R = 0{\cdot}0001$ and Q-unit matrix. The time increment chosen was $\Delta t = 0{\cdot}6$ s. The results computed without the control and by the gradient method are compared in Figure 6. The results shown by solid lines are those by the gradient method and the results marked as circles are those without the control. By controlling the discharge at the outlet as shown in Figure 6(c), the water elevation upstream



(a) Discharge of inflow at point A

(d) Water elevation at point A

(b) Discharge at point B

(e) Water elevation at point B

(c) Discharge of outflow at point C

(f) Water elevation at point C

Figure 7

and downstream can be controlled as shown in Figures 6(d)–6(f) at sites A, B and C respectively. It is seen in Figure 6 that an almost flat water elevation has been obtained. The results computed by the Sakawa–Shindo method and by the gradient method are compared in Figure 7. The results shown by solid lines are those by the gradient method and the results marked as circles are those by the Sakawa–Shindo method. In Figures 7(c) and 7(f) the computed results of the discharge and water elevation respectively at point C obtained by the present method are shown in comparison with the results obtained by the Sakawa–Shindo method in which the duration time of the control (T_f) is assumed to be coincident with the terminal time of the flood (t_f). Both sets of results are completely coincident for both discharge and water elevation. The computed duration time of the control is much shorter than the terminal time of the flood. The practical running time of computation is wholly dependent on the duration time of the control. Thus practical computation time can be saved by the present method.

8. CONCLUSIONS

An optimal control method for flood control has been presented taking into account the fact that the duration of the control is not necessarily coincident with the duration of the flood. The terminal time of the control is computed by the terminal condition that the water elevation and control discharge at the terminal state should be zero. The computed water elevation and velocity are the same as those obtained by the tracking control technique presented in previous papers. The computed terminal time is much shorter than the terminal time of the flood. Thus the practical running time of computation can be markedly reduced by the present method.

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REFERENCES

1. V. M. Ponce, 'Linear reservoirs and numerical diffusions', *Proc. ASCE*, **106**, (HY5), 691–699 (1980).
2. V. M. Ponce and A. J. Tsivoglou, 'Modeling gradual dam breaches', *Proc. ASCE*, **107**, (HY7), 829–838 (1981).
3. A. O. Akan and B. C. Yen, 'Diffusion-wave flood routing in channel networks', *Proc. ASCE*, **107**, (HY6), 719–732 (1981).
4. B. Hunt, 'A symptotic solution for dam break on sloping channel', *Proc. ASCE*, **109**, (HY12), 1698–1706 (1983).
5. D. A. Hughes and H. C. Murrell, 'Non-linear run off routing—a comparison of solution methods', *J. Hydro. Eng.*, **85**, 339–347 (1986).
6. C. V Bellos and J. G. Sakkas, '1-D dam-break flood-wave propagation on dry bed', *Proc. ASCE*, **113**, (HY12), 1510–1524 (1987).
7. F. Wakamori, S. Masui, M. Funabashi and M. Ohnari, 'Optimal operation of a multireservoir during flood periods', *Control Science and Technology (8th Triennial World Congr.)*, Kyoto, 1981, pp. 3835–3840.
8. R. B. Buchanan and R. L. Bras, 'A study of real time adaptive closed-loop reservoir control algorithm', in *Optimal Allocation of Water Resources, IAHS Publ. 135*, 1982, pp. 317–326.
9. M. Papageorgiou, 'Optimal multireservoir network control by the discrete maximum principle', *Water Resources Res.*, **21**, 1824–1830 (1985).
10. J. Muskatirovic and R. Kapor, 'Analysis of the control of floods caused by the failure of a cascade system of dams', *Proc. 2nd Int. Conf. on the Hydraulics of Floods and Flood Control*, 1985, pp. 49–61.
11. Y. Tanaka and M. Kawahara, 'Control of flood using finite element method and optimal control theory', in T. J. Chung and G. R. Karr (eds), *Finite Element Analysis in Fluids*, University of Alabama in Huntsville Press, Huntsville, AL, 1989, pp. 617–622.
12. T. Kawasaki and M. Kawahara, 'A flood control of dam reservoir by conjugate gradient method and finite element method', in T. J. Chung and G. R. Karr (eds), *Finite Element Analysis in Fluids*, University of Alabama in Huntsville Press, Huntsville, AL, 1989, pp. 629–634.

13. M. Kawahara and T. Kawasaki, 'A flood control of dam reservoir by conjugate gradient and finite element methods', *Proc. 5th Int. Conf. on Numerical Ship Hydrodynamics*, Hirosima, 1989, pp. 45–56.
14. T. Umetsu, Y, Tanaka and M. Kawahara, 'Optimal control of flood using finite element method', *Proc. JSCE*, **9**, (1), 11s–23s (1992).
15. Y. Shimada, M. Kawahara and T. Umetsu, 'Adaptive control method to operate the water gate in dam as the practical problem', *Proc. Fourth Int. Conf. on Computing in Civil and Building Engineering*, Tokyo, 1991, p. 324.
16. Y. Shimada, M. Kawahara and T. Umetsu, 'Adaptive control to operate water gate of dam', *Proc. Asian Pacific Conf. on Computational Mechanics*, Hong Kong, 1991, pp. 1749–1754.
17. M. Kawahara and Y. Shimada, 'Optimal control with constraint on the control value to operate water gate of dam', *Proc. Finite Elements in Fluids, New Trends and Applications*, Barcelona, 1993, pp. 1272–1280.